Nominal Rocket Flight

Fuel Needed to Achieve Orbit

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Introduction

Although humans have been exploring space for 50 years, the science of rockets and space travel has yet to be perfected. Just recently, a Russian rocket failed to deliver satellites into orbit. A model of a rocket launch is useful for both verifying whether a launch will work and evaluating alternative launch methods, namely electromagnetic launching.

The questions we hope to answer with our model are "What is the optimal amount of fuel needed to bring a particular payload into orbit around Earth and what is the efficiency of an electromagnetic launch mechanism?" To do so, we modeled the flight of a rocket from lift-off to steady orbit.

Assumptions

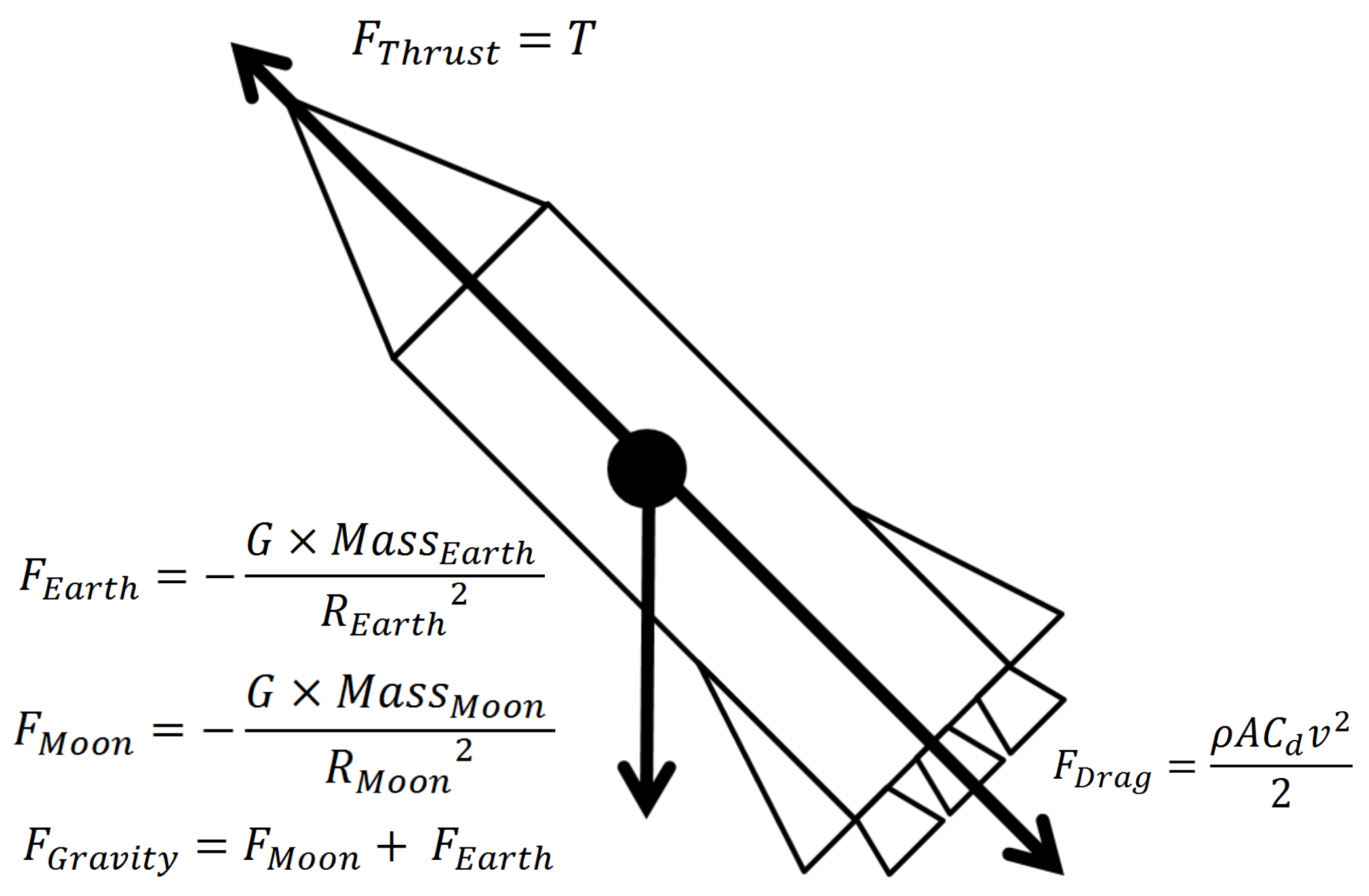
- Only gravity of Earth and Moon are modeled
- No supersonic motion
- Fuel burns steadily
- Constant thrust
- No wind

Model

The model of the rocket trajectory included three forces: gravity, drag, and thrust.

The desired direction of thrust was determined by the difference between desired total energy and current total energy. The desired total energy was divided into kinetic energy (tangent to the orbit) and potential (perpendicular to the orbit).

The fuel needed to get into orbit for a particular payload was found by making an initial guess (high enough to get the rocket to orbit) then reducing fuel until the point right before the rocket no longer reaches orbit. The energy efficiency was modeled by calculating the total kinetic energy of the rocket at launch then adding the chemical potential energy of the rocket fuel. The amount of rocket fuel is minimized to being only what's necessary for getting into orbit.



$$F_{Thrust} = T$$

$$F_{Earth} = -\frac{G \times Mass_{Earth}}{{R_{Earth}}^2}$$

$$F_{Moon} = -\frac{G \times Mass_{Moon}}{{R_{Moon}}^2}$$

$$F_{Gravity} = F_{Moon} + F_{Earth}$$

$$F_{Drag} = \frac{\rho A C_d v^2}{2}$$

ρ = Air density

A = Frontal surface area of the rocket

C_d = Coefficient of drag

v = Velocity of the rocket

G = Gravitational constant

R_{Earth} = Distance between the rocket and the Earth

R_{Moon} = Distance between the rocket and the Moon

T = Constant thrust value

Results

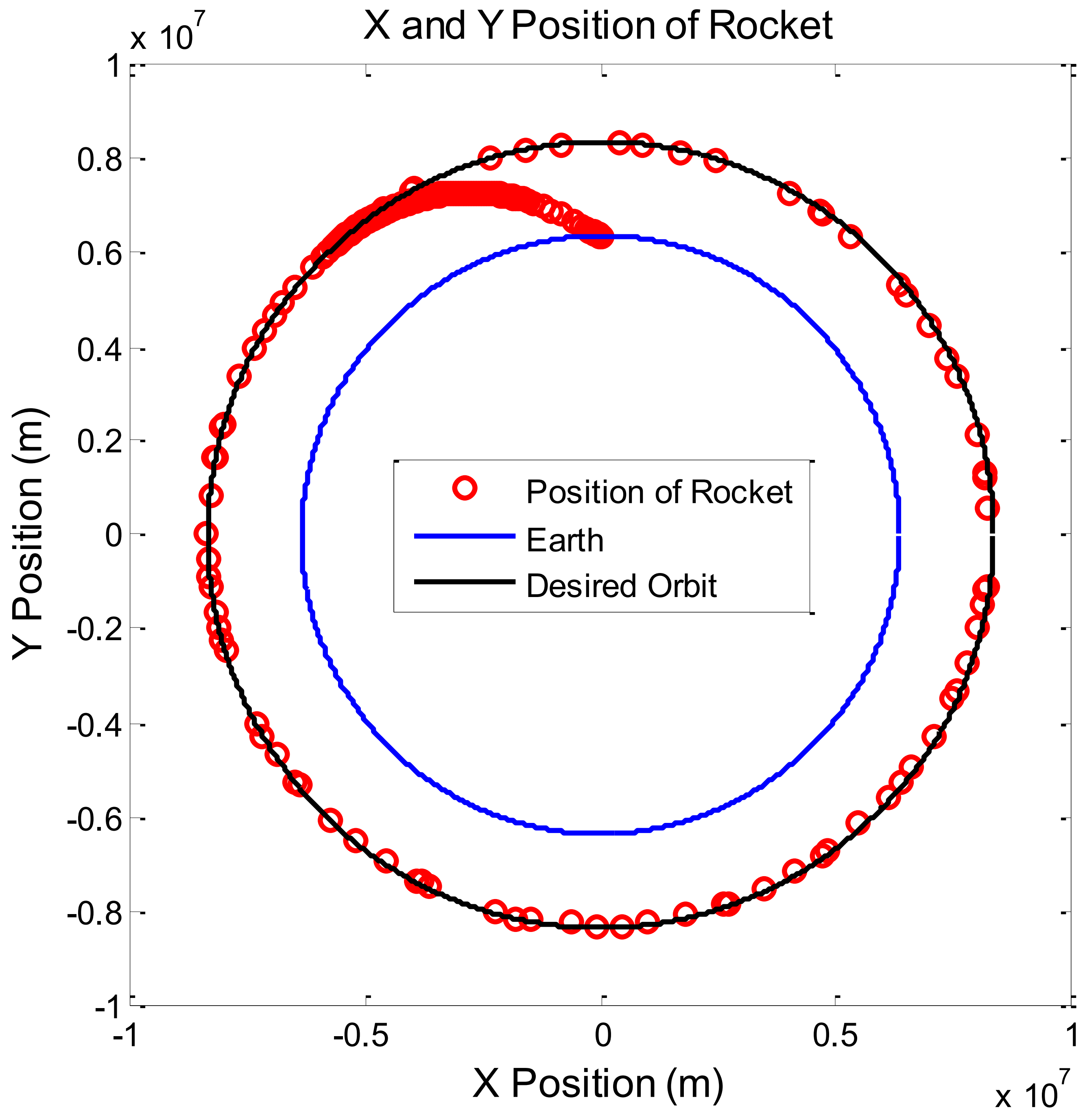


Figure 1: Flight of the rocket. The orbit radius of this flight is 2e6 m from the surface of the Earth. When the rocket reaches orbit, the thrust is cut of and it begins traveling in a steady orbit. In order for the orbit to be steady, the velocity must be enough that the centripetal acceleration cancels out the gravitational forces on the rocket.

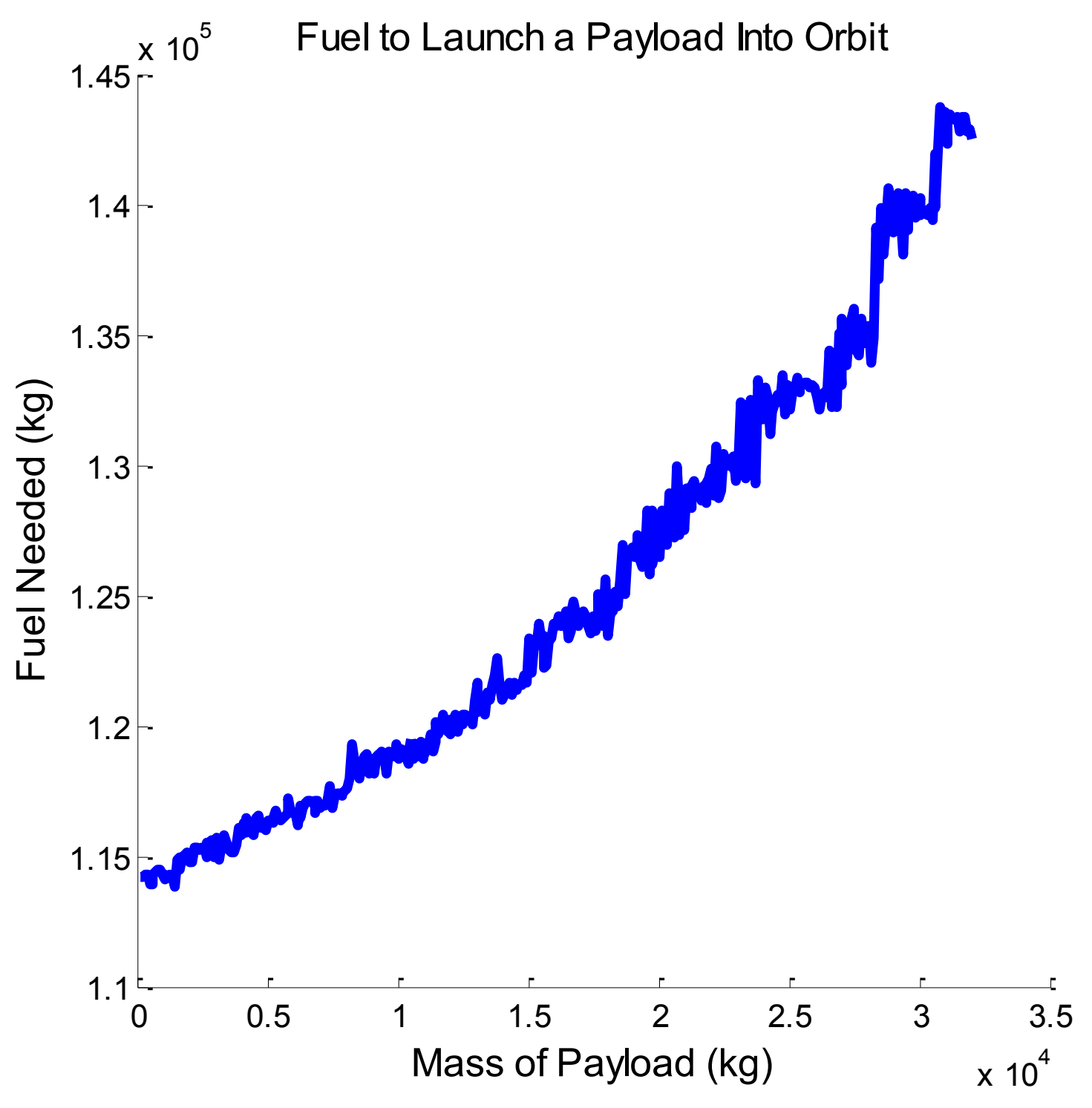


Figure 3: Minimum fuel needed to launch a payload of a given mass into orbit. The slope is increasing.

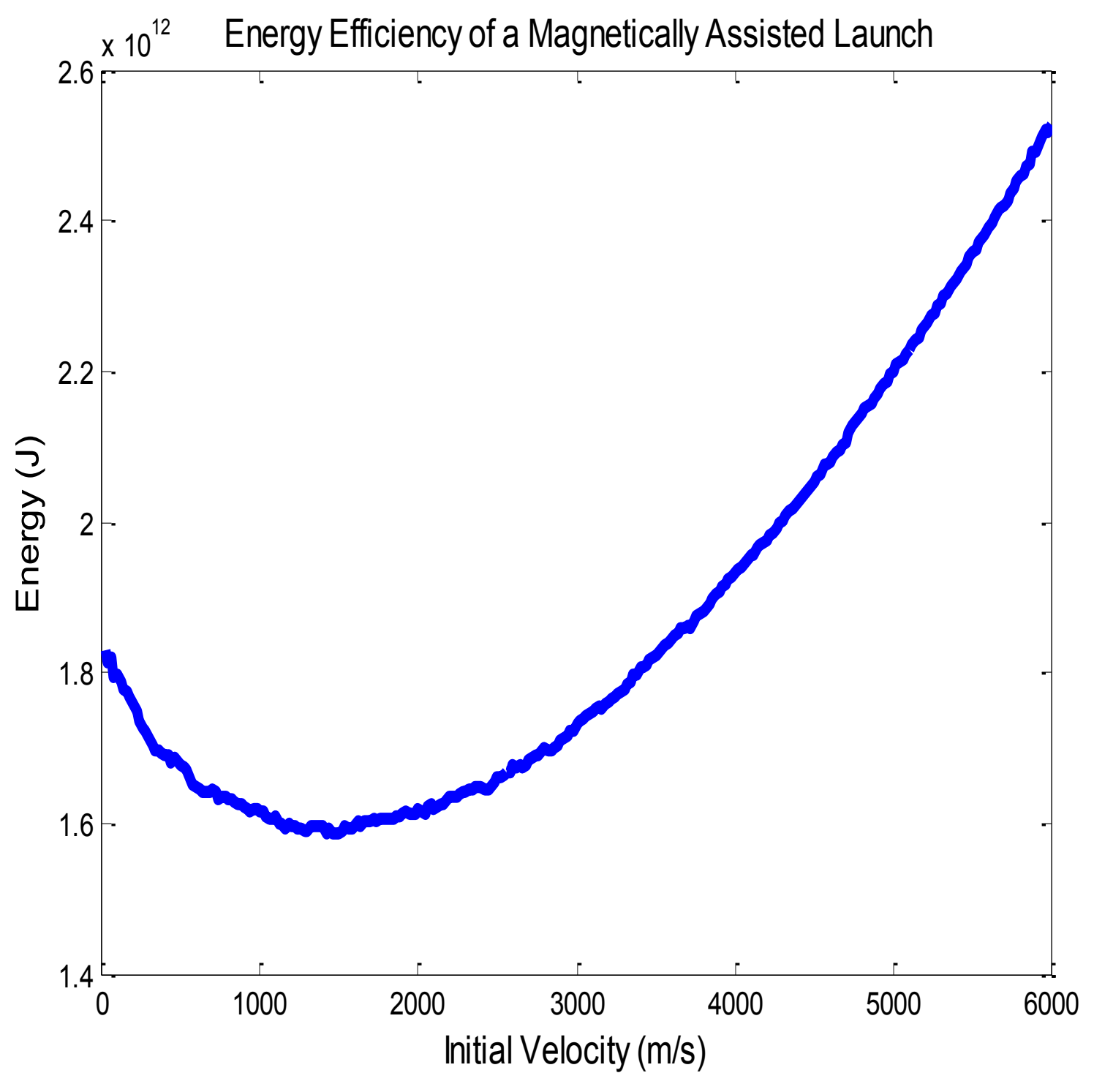


Figure 4: Energy efficiency of a magnetically assisted launch. The minimum at approximately 1500 m/s is the optimal initial velocity.

Validation

Given a particular thrust, the rocket would go into circular orbit. Given less than that amount of thrust, the rocket would crash into the Earth (See Figure 5). The mass, thrust, diameter, and burn time are also based on real values and are all within 2 orders of magnitude of a Delta II rocket [2].

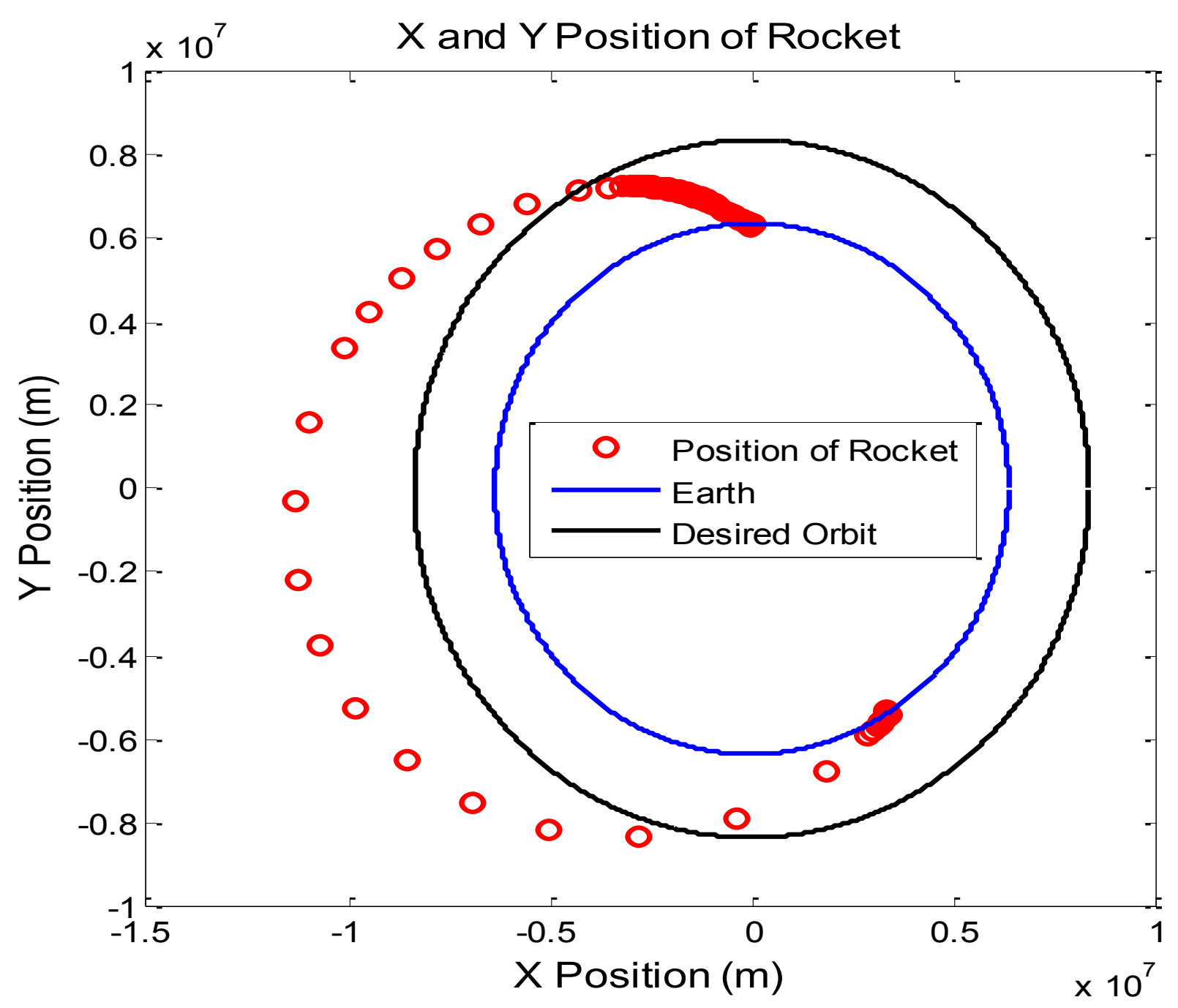


Figure 5: Flight of the rocket when the thrust is too low (0.75% of the needed thrust). The rocket crashes into the Earth.

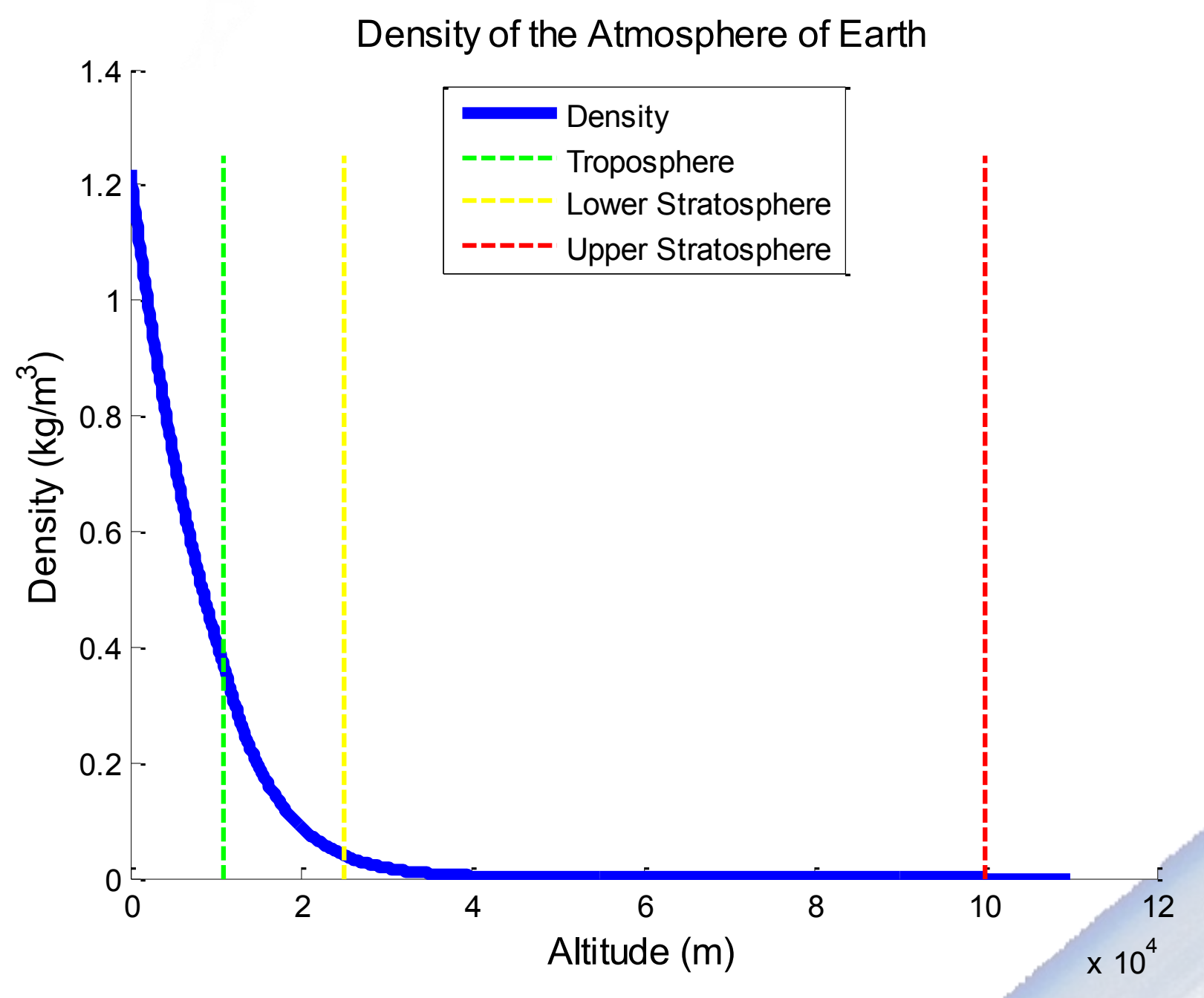


Figure 6: The modeled density of air in the atmosphere. Data for this model is from [1]. At 100 km, the density is assumed to be zero.

The model successfully simulated physical properties when we ran simplified tests. For example, when we plotted the density of the atmosphere over altitude, it acted as expected.

SpaceX Rocket Comparison

	SpaceX	Model	Error (%)
Mass of Fuel (x 9) (kg)	42000	42000	0
Thrust (x 9) (N)	6.16E6	7.5E6	21.75
Fuel Rate (x 9) (kg/s)	140	100	28.57
Altitude of Orbit (m)	294500	294500	0
Diameter Nose Cone (m)	3.6	3.6	0
Distance (t = 4:30) (m)	210000	291120	38.63
Velocity (t = 4:40) (m/s)	3700	7602	105.48
Distance (t = 6:00) (m)	264000	301500	14.20
Velocity (t = 6:00) (m/s)	4300	7232	68.2
Distance (t = 7:20) (m)	300000	312470	4.16
Velocity (t =7:20) (m/s)	5500	6809	23.81

Table 1: Comparison between flight data of the SpaceX rocket launch on December 8 and a simulated trajectory. Using similar parameters, the results are within one order of magnitude of the actual values. Differences can be attributed to simplifications in the model, mainly that the modeled rocket has one stage and the SpaceX rocket has two.

Conclusion

For a rocket of the parameters we specified, we determined the specific amount of fuel needed to successfully launch a payload into orbit. We also determined the usefulness of an electromagnetic launch and the optimal initial launch velocity for a given mass, highlighting the importance of pursuing alternative launch methods for the future of space travel. However, this model is limited in scope and would need to be substantially expanded for use of an actual rocket.

Future Work

- Multistage rocket
- Supersonic drag
- Launching to/orbiting another planet

References

Thanks to Alisha Sarang-Sieminski, Mark Somerville, Allen Downey, and Sanjoy Mahajan

[1]www.grc.nasa.gov/WWW/K-12/airplane/atmosmet.html
[2]http://www.spacelaunchreport.com/delta2.html